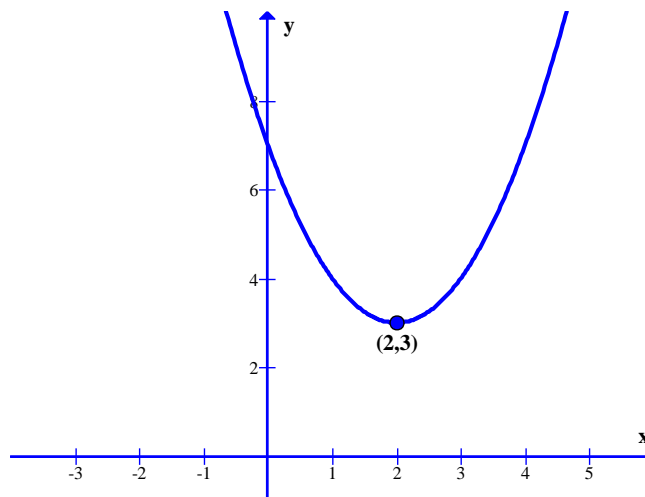


§ 2.3: Increasing and Decreasing Functions; Average Rate of Change

Increasing and Decreasing Functions

It is very useful to know where the graph of a function rises and where it falls. The graph shown below falls then begins to rise:



When the graph of a function is falling, we say the function is decreasing; when the graph rises, we say the function is increasing. The above graph is the graph of the function $f(x) = (x - 2)^2 + 3$. Thus we say that f is decreasing from $(-\infty, 2)$ and f is increasing from $(2, \infty)$.

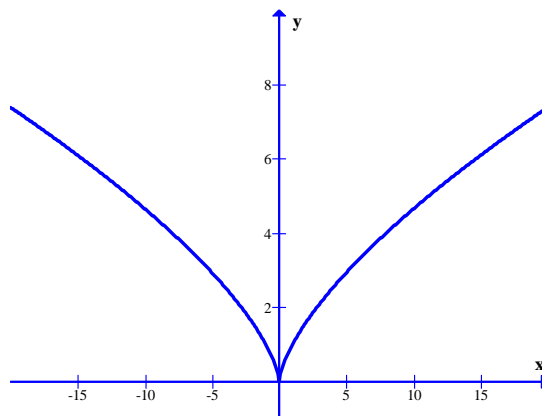
Definition of Increasing and Decreasing Functions

f is **increasing** on an interval I if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I .

f is **decreasing** on an interval I if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ in I .

Example 1	Using a Graph to Find Intervals where a Function Increases and Decreases
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The following graph is the graph of the function $f(x) = x^{2/3}$. Find the domain and range of then function, then find the intervals on which f is increasing and decreasing.



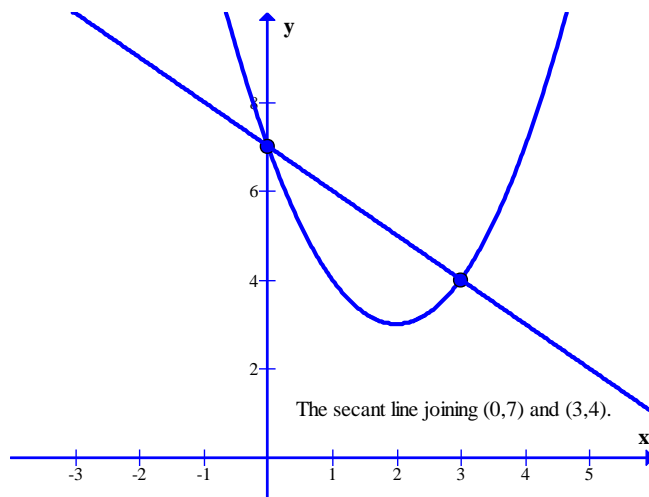
Average Rate of Change

Average Rate of Change

The **average rate of change** of the function $y = f(x)$ between $x = a$ and $x = b$ is

$$\text{average rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{f(b) - f(a)}{b - a}$$

The average rate of change is the slope of the **secant line** between $x = a$ and $x = b$ on the graph of f . The secant line is the line that passes through the points $(a, f(a))$ and $(b, f(b))$.



Example 2	Calculating the Average Rate of Change
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For the function $f(x) = (x - 2)^2 + 3$, find the average rate of change between the following points:

- (a) $x = 1$ and $x = 3$ (b) $x = 0$ and $x = 3$

Compare your answer to part (b) with the slope of the line sketched in the above graph.

Example 3	Average Speed of a Falling Object
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If an object is dropped from a tall building, then the distance it has fallen after t seconds is given by the function $d(t) = 16t^2$. Find its average speed over the following intervals:

- (a) Between 1 s and 6 s.
- (b) Between $t = a$ and $t = a+h$, where $h \neq 0$.

Example 4	Linear Functions Have Constant Rate of Change
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Let $f(x) = 4x - 6$. Find the average rate of change of f between the following points.

(a) $x = 1$ and $x = 7$

(b) $x = -4$ and $x = 4$

(c) $x = 9$ and $x = 100$

(d) $x = 1$ and $x = a+h$, where $h \neq 0$.

What do you notice about the average rate of change?

Homework

Due: _____

6 – 12 (even), 18 – 30 (even)